Control System Design
Lecture 3

Associate Prof. Dr. Klaus Schmidt

Department of Mechatronics Engineering – Çankaya University

Elective Course in Electronic and Communication Engineering
Credits (2/2/3)

Webpage: http://ECE441.cankaya.edu.tr

Nonlinear System Modeling: Remarks

**LTI System Operators**
- Proportional gain
- Differentiation
- Integration
- Lead/lag components
- Summations

⇒ All linear operators can be represented by transfer functions

**Nonlinear Systems**
- Contain nonlinear system operators

⇒ No transfer function representation
Nonlinear System Modeling: Magnetic Suspension

Schematic

Simplified Description

Simplifications

- Sphere represents vehicle
- Magnet represents suspension system

Klaus Schmidt
Department of Mechatronics Engineering – Çankaya University

Nonlinear System Modeling: Equations

Computation

Gap 1

Klaus Schmidt
Department of Mechatronics Engineering – Çankaya University
Nonlinear State Equations: General Form

**State Equations**
\[
\dot{x} = f(x, u, w) \\
y = h(x, u)
\]

**Notation**
- state: \(x\), output: \(y\), input: \(u\), disturbance \(w\)
- \(f\): continuous in \(x\), \(u\), \(w\) and additional assumptions (see for example ECE 564)
- \(h\): continuous in \(x\), \(u\)

**Example**

---

Klaus Schmidt  
Department of Mechatronics Engineering – Çankaya University
Nonlinear System Modeling: Remarks

**Synthesis and Analysis Techniques for Nonlinear Systems**

- Beyond the scope of this lecture
  → Master-level course ECE 564
- Extensive literature

**Set-point Linearization**

- Consider system behavior in the vicinity of a given set-point
  → Assume almost linear behavior close to the set-point
  → Find a linear system model to approximate the nonlinear system

---

**Set-Point: Definition**

**Set-point Definition**

*A set point is a stationary (non-changing) state of a system where the system output maintains a constant set-point value $y_{SP}$*

**Computation of a Set-point**

- Given: $y_{SP}$, $w_{SP}$
- We want to compute $x_{SP}$ (constant set-point value of the state) and $u_{SP}$ (constant set-point value of the input)
- Computation

\[
y_{SP} = h(x_{SP}, u_{SP}) \\
0 = \dot{x} = f(x_{SP}, u_{SP}, w_{SP})
\]

⇒ Solve for $x_{SP}$, $u_{SP}$
Set-Point: Example

Magnetic Suspension

Set-point Linearization: Description

Explanation

- Compute a “small signal” approximation of the nonlinear system that is valid close to the set-point
- Introduce “Difference variables“ (deviation from the set-point)
- $\Delta x = x - x_{SP}$, $\Delta y = y - y_{SP}$, $\Delta u = u - u_{SP}$, $\Delta w = w - w_{SP}$

Taylor Series Expansion

$\Delta \dot{x} \approx f(x_{SP}, u_{SP}, w_{SP}) + \frac{\partial f}{\partial x}\bigg|_{SP} \Delta x + \frac{\partial f}{\partial u}\bigg|_{SP} \Delta u + \frac{\partial f}{\partial w}\bigg|_{SP} \Delta w$

$= A \Delta x + b \Delta u + o \Delta w$

$\Delta y \approx h(x_{SP}, u_{SP}) - y_{SP} + \frac{\partial h}{\partial x}\bigg|_{SP} \Delta x + \frac{\partial h}{\partial u}\bigg|_{SP} \Delta u = c^T \Delta x + d \Delta u$
Set-point Linearization: Example

Example Equations

\[ \dot{x}_1 = x_2 \]
\[ \dot{x}_2 = -g + \frac{K_M}{m} \frac{u^2}{(d - x_1)^2} - \frac{1}{m} w \]
\[ y = x_1 \]

Computation

Klaus Schmidt
Department of Mechatronics Engineering – Çankaya University
Set-point Linearization: Magnetic Suspension Example

Linearized State Equations

Task
- Characterize nonlinear system behavior close to a set-point

Method
- Write system representation in terms of “difference variables”
- Use first-order Taylor series approximation for nonlinearities

Result
- We get a linear system model for the nonlinear system
- Linear methods can be used for the nonlinear system close to the set-point
- Important restriction
  → Linear model is only valid in the vicinity of the set-point